A Macroscopic Model for Multi-Robot Stigmergic Coverage (Extended Abstract)

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ABSTRACT

This paper explores a multi-robot coverage approach called **StiCo** (for "Stigmergic Coverage") by deriving a probabilistic macroscopic model. The proposed model makes it possible to quickly and efficiently study the swarm-type behavior of **StiCo**, and also allows for making predictions about its long term behavior. The model is validated in a twofold way: through computer simulations, and with real robots.

Categories and Subject Descriptors

I.2.11 [**Artificial Intelligence**]: Distributed Artificial Intelligence—*Multiagent systems*; I.2.9 [**Artificial Intelligence**]: Robotics

General Terms

Theory, Verification

Keywords

Swarm Robots, Environment Coverage, Stigmergic Communication, Macroscopic Probabilistic Models

1. INTRODUCTION

StiCo is specifically designed for robots equipped with low-range sensors that operate in environments where direct robot-robot communication is limited or not possible at all [1]. The basic notion underlying this approach is to partition the environment into equal circular regions (also called territories) where each robot takes responsibility to guard one of these regions. Therefore, **StiCo** answers the core question "*How should robots move in order to decrease the intersections of their territories*".

In **StiCo**, each robot starts to move with a constant forward linear velocity, and a constant angular velocity, which results in a circular motion on the borders of the robot's territory. The forward linear velocity remains constant during the whole mission. However, when the robot sensor detects a pheromone (i.e. an evaporable robot trail), it indicates to the robot that it is about entering another territory, and therefore the robot changes its circling direction immediately. In this way, the robot establishes its territory in a new region without any intersection with the previous territory. The effective coverage behavior of **StiCo** is shown in [2] by means of various simulation scenarios for robotic swarms of different sizes. This behavior is also validated with experiments on real robotic swarms[1].

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2. MACROSCOPIC MODEL DESIGN

Consider the number of singular territories (i.e., the territories, which does not have intersection with any other one) as the performance criterion for **StiCo**. In order to be able to capture this criterion formally, the following three simplifying (but not unrealistic) assumptions are made:

Assumption 1. *In each iteration, just one territory leaves its x-tuple group ($x > 1$), where an n-tuple group is a cluster of n territories which can not be separated into two disjoint clusters.*

Assumption 2. *Any n-tuple group ($n > 1$) of territories have a configuration similar to a hexagon tessellation in which territories are inscribed in hexagons.*

Assumption 3. *The free area required for a robot to establish its new singular territory after leaving its current group is called landing region and is approximated as $S_L = 0.5\pi(2R)^2$.*

Let's define the state $C^n, n = 1, 2, \ldots, M$, for the case that there are n singular territories in the environment. Then, we need a mathematical expression to compute the probability of transition from state C^{n1} to state C^{n2}, in one iteration (Assumption 1). This probability is denoted by $P_{n1,n2}$.

The first step for computing probability $P_{n1,n2}$, is to partition a general state C^n to all of its possible configurations (the word *partition*, refers to a concept of number theory). The configuration $C^n_{T_{a1},T_{a2},\ldots,T_{ak}}$ denotes a configuration in state C^n, in which T_{ai} denotes existence of one ai-tuple in the configuration. If we define $Q^n(K)$ as the probability that a swarm of territories be in state C^n in K-*th* iteration, then the discrete state transition model can be written as

$$\begin{bmatrix} Q^1(K+1) \\ Q^2(K+1) \\ \vdots \\ Q^M(K+1) \end{bmatrix} = \begin{bmatrix} P_{1,1} & \cdots & P_{M,1} \\ \vdots & \ddots & \vdots \\ P_{1,M} & \cdots & P_{M,M} \end{bmatrix} \begin{bmatrix} Q^1(K) \\ Q^2(K) \\ \vdots \\ Q^M(K) \end{bmatrix} \quad (1)$$

For computing $P_{n,n-1}$, which is the transition from C^n to C^{n-1}, the chance that a singular territory becomes a member of a double group should be computed. Let $L(M, n)$ be

[1] http://swarmlab.unimaas.nl/stico/

a function that computes number of possible configurations of M territories, in which exactly n of them are singular. Then, consider the t-th configuration of C^n as

$$C^n_{\underbrace{T_1,T_1,\ldots,T_1}_{n},\underbrace{T_2,T_2,\ldots,T_2}_{r_t},T_{a1_t},T_{a2_t},\ldots,T_{ak_t}} \tag{2}$$

The probability for a transition from the t-th configuration of C^n to one of the configurations of C^{n-1} is computed by calculating the probability that one of the non-singular territories leave their group and intersect with one of the n singular territories:

$$p^t_{n,n-1} = \frac{M-n-r_t}{M-n} \times n\frac{S_L}{A-A_o}\left(1-\frac{S_L}{A-A_o}\right)^{n-1} \tag{3}$$

where A_o denotes the region occupied by non-singular territories: $A_o = (M-n)S_H$, and S_H denotes the area of a hexagon circumscribed by a territory: $S_H = \frac{3\sqrt{3}}{2}R^2$ (Assumption 2). S_L is the area of landing region approximated in Assumption 3.

Therefore, the overall probability function $P_{n,n-1}$ is defined as

$$P_{n,n-1} = \sum_{t=1}^{L(M,n)} p^t_{n,n-1} \tag{4}$$

For computing the chance of transition from C^n to C^{n+1} which is for the case that number of singular territories increases by one, consider the same t-th configuration provided in (2). we can calculate $p^t_{n,n+1}$ as

$$p^t_{n,n+1} = \frac{M-n-r_t}{M-n} \times n\left(1-\frac{S_L}{A-A_o}\right)^{n} \tag{5}$$

and overall probability function $P_{n,n+1}$ is defined as

$$P_{n,n+1} = \sum_{t=1}^{L(M,n)} p^t_{n,n+1} \tag{6}$$

For $P_{n,n+2}$ there is just one configuration in which a transition from C^n to C^{n+2} happens: A territory leaves a double group, and instead becomes a singular territory. In this way, two new singular territories will be added to the previous configuration. For the t-th configuration provided in (2), we have

$$p^t_{n,n+2} = \begin{cases} \frac{r_t}{M-n} \times n\left(1-\frac{S_L}{A-A_o}\right)^{n} & r_t > 0 \\ 0 & r_t = 0 \end{cases} \tag{7}$$

and overall probability function $P_{n,n+2}$ is defined as

$$P_{n,n+2} = \sum_{t=1}^{L(M,n)} p^t_{n,n+2} \tag{8}$$

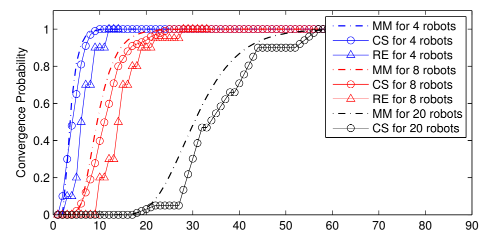
Finally, if we ignore the probability for transition from C^n state to the states C^i, in which $i < n-1$ or $i > n+2$, then the probability for remaining in the same state is

$$P_{n,n} = 1 - P_{n,n+1} - P_{n,n+2} - P_{n,n-1} \tag{9}$$

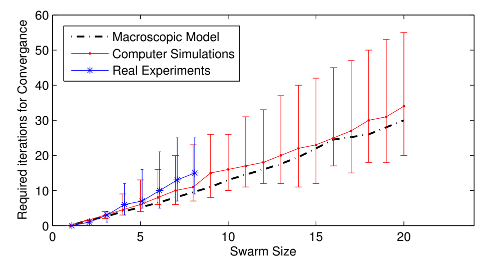
In order to check the conditions of fundamental Ergodic Theorem for Markov chains on P matrix, these conditions are simply explained as: (1) P should be stochastic: The values of P must be within the range $[0,1]$ and each column (or row) sums to 1. (2) P should be irreducible: From each state of our system, it must be possible to get to any other state. (3) P should be aperiodic: The graph represented by P should not be bipartite. The first condition holds based on the fact that each probability is in the range of $[0,1]$, and Eq. (9) which shows each column sums to 1. The two other conditions can be easily checked with constructing the graph represented by P. Therefore, P is a Markov chain which can denote a stationary configuration $\Pi = \lim_{i\to\infty} P^i.Q(0)$, where $Q(0)$ can be any initial probability distribution for initial configuration.

3. RESULTS

Three groups of 4, 8, and 20 robots are initialized at the center of environment. For each group, the probability of being in the final stationary configuration, $Q(.)$, is first computed using the macroscopic model. Then computed by using computer simulations, and finally by using real robot experiments. The results of computing the convergence probability are illustrated in Fig. 1a. The presented results show that the macroscopic model can estimate the behavior of **StiCo** for robotic swarms of various sizes. As shown in Fig. 1b, the convergence speed of **StiCo** increases linearly with growth of the swarm population.



(a)



(b)

Figure 1: Model verification (a) Convergence probability in different iterations (MM: Macroscopic Model, CS: Computer Simulations, RE: Real Experiments). (b) Effects of swarm size on convergence time.

4. REFERENCES

[1] B. Ranjbar-Sahraei, G. Weiss, and A. Nakisaee. A multi-robot coverage approach based on stigmergic communication. In *Multiagent System Technologies*, volume 7598 of *Lecture Notes in Computer Science*, pages 126–138. Springer, 2012.

[2] B. Ranjbar-Sahraei, G. Weiss, and A. Nakisaee. Stigmergic coverage algorithm for multi-robot systems (demonstration). In *Proceedings of the 11th International Conference on Autonomous Agents and Multiagent Systems-Volume 3*, pages 1497–1498, 2012.